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STRAIN MEASUREMENT ERRORS IN MATERIALS OF LOW MODULUS

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ABSTRACT

When conventional SR-4 gages are bonded to the surfaces of materials which have a very low modulus of elasticity, the local stiffening effect of the gage may be of great importance. Not only is the recorded strain less than the value of strain which would exist if the gage were not present, but the distribution of strain in the material near the gage is altered. These effects are shown by examples worked out for a common SR-4 type gage on rubber and other materials with low E. The design and operating characteristics of several bonded gages which have been developed to reduce this localized stiffening are presented.

INTRODUCTION

In most techniques for measuring strain in test specimens and parts made of rubber-like materials, a relatively large change in dimension over a sizeable gage length is recorded. These measurements are most commonly made with dividers, dial indicators, or lever systems, because large strains are permissible and expected. But in testing rubber and rubber-like materials subjected to vibratory or impact loads the strains of interest may not be large, and the strain pickup must have rapid response and low inertia. In addition, the development within the past 15 years of indicating and recording equipment designed specifically for use with resistance type strain gages makes it desirable to use the new type of gage whenever possible.

Because of the low inertia response needed and the new developments, the ideal strain gage for use on rubber and rubber-like materials appears to be a bonded resistance-type gage having the following special characteristics:

1. An effective modulus of elasticity so low that the stiffening effect of the gage need not be considered.
2. The ability to follow large strains and subsequently re-zero.

Limitations on the Use of SR-4 Gages

The SR-4 type of bonded resistance gages have proven so adaptable that as a first step it seemed logical to investigate the special characteristics of the SR-4 gage referred to in the Introduction. In fact, since a Post-Yield* gage is available which will follow strains up to ten percent it seemed necessary only to investigate the stiffening effect of the SR-4 gages.

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*The PA-3 Post-Yield Strain Gage is available from the Baldwin-Lima-Hamilton Corporation.

The cross-section of a typical mounted SR-4 is shown in figure 1. The entire gage including the bond between the gage proper and the surface on which it is mounted is obviously not a homogeneous solid. Therefore in defining its stiffness in terms of a modulus of elasticity E it is convenient to work in terms of an effective modulus for the entire gage. Due to differences in spacing of the grid wires, the type of grid construction, flat or wrap-around, the kind of grid wire, Advance or Iso-Elastic, and the type of bonding cement, nitro-cellulose or phenol-resin, the effective modulus of elasticity would be expected to differ among the various gage types. In fact, the effective modulus of elasticity of gages of the same type will vary somewhat due to different thickness of bond obtained in their mounting. To determine how the stiffness of the SR-4 gages may limit their usefulness, only a single gage type need be investigated.

The gage type chosen for this investigation was the A-1. Two A-1 gages were trimmed to the form shown in figure 2 and cemented back to back with nitro-cellulose cement. These two gages were connected electrically as opposite arms of a Wheatstone bridge; the other two arms of the bridge consisted of two additional gages of the same lot number also cemented back to back. All readings were taken on a K-unit which had been modified for connection to a four arm bridge. The ends of the two gages to be tested were clamped between small steel plates and loaded in tension by means of dead weight (figure 2). Figure 3 is a plot of the load per gage versus the measured values of strain produced by that load. From figure 3 the force per unit of strain is evaluated as:

$$\left(\frac{F}{e}\right)_{\substack{\text{Type} \\ \text{A-1 Gage}}} = 1{,}040 \quad \text{lb/in/in}$$

in which F = force in lb.

and e = strain in in/in.

Since

$$E = \frac{s}{e} = \frac{F}{A\,e},$$

we may write

$$(EA)_{\substack{\text{Type} \\ \text{A-1 Gage}}} = \frac{F}{e} = 1{,}040 \quad \text{lb/in/in}$$

in which E = effective modulus of elasticity of the entire gage

and A = the cross sectional area of the equivalent homogeneous gage.

The importance of this product EA, which is the stiffness of the gage, can best be illustrated by an example.

Consider a tensile specimen having a rectangular cross section of A_s. The force required to produce a unit strain is

$$\frac{F_s}{e_s} = A_s\,E_s \qquad (1)$$

in which the subscript s is used to denote the specimen. Now if we mount an SR-4 gage on each edge of this specimen, as shown in figure 4, and assume that at the cross section where the gages are mounted the strain is uniform

($\underline{e}$ of gages = $\underline{e}$ in the specimen) the force required to produce a unit strain may be determined as follows:
Since

$$e_g = e_s$$

$$e_{g,s} = \frac{F_g}{(EA)_g} = \frac{F_s}{E_s A_s} \tag{2}$$

in which the subscript g denotes the gages. The total force applied is equal to the sum of the resistive forces developed by the specimen and by the gages, i.e.

$$F_{Total} = F_g + F_s;$$

and substituting (2) into this expression gives

$$\frac{F_{Total}}{e_s} = (EA)_g + E_s A_s. \tag{3}$$

The ratio of force required to produce a given strain with gages in place to the force required when no gages are present is

$$R = \frac{(F/e)_{With\ Gages}}{(F/e)_{Without\ Gages}} = \frac{(EA)_g + E_s A_s}{E_s A_s} = \frac{(EA)_g}{E_s A_s} + 1 \tag{4}$$

This ratio is a measure of the stiffening or reinforcing effect of the gages. The value of this effect, and hence the possible error introduced in strain measurement, is obviously dependent upon $\frac{(EA)_g}{E_s A_s}$.

We can determine the reinforcing effect of two type A-1 gages trimmed to a width of 0.375 inches and bonded with nitro-cellulose cement on opposite sides of specimens 0.375 inches wide (see figure 4), since the stiffness of a type A-1 gage trimmed to a width of 0.375 inches and bonded with nitro-cellulose cement has been established. For a type A-1 gage,

$$(EA)_g = 1{,}040 \text{ lb/in/in};$$

then, for two such gages,

$$(EA)_g = 2{,}080 \text{ lb/in/in}.$$

With this value, (4) has been solved for various materials having various thicknesses, and the results are tabulated in Table I. Figure 5 is a nomograph for the solution of (4), which enables the reinforcing effect of two A-1 gages to be determined rapidly.

Although SR-4 gages can be mounted on very thin metal sections without introducing any appreciable reinforcing effect, the reinforcing effect may be large when they are mounted on thin plastic sections or on rubber-like materials of any size.

The assumption that the strain is uniform through the cross sections at which the gages are applied is certainly an over-simplification, even for tensile loading. The localized stiffening of the surface will in fact produce distortion of the strain pattern within the specimen, and this distortion will be more complex when the shape of the specimen and/or the type of loading becomes more complex. The distortion of the strain pattern in the vicinity of the gages makes the entire problem of gage reinforcing more difficult because the distortion makes it more difficult to apply any meaningful correction factor to values of strain measured by the gages.

Table I. The Reinforcing Effect of Two Type A-1, SR-4 Gages Bonded to the Opposite Sides of Tensile Specimen of Various Materials and Sizes.

Material	E_s (psi)	Size (t x w, fig.4) (in x in)	Reinforcing Effect = $\frac{(F/e)_{With\ Gages}}{(F/e)_{Without\ Gages}}$
			Two A-1 Gages Bonded With Nitro-Cellulose
Rubber or Rubber-Like	1,000	1 x 3/8	6.55
	5,000	1 x 3/8	2.11
Plastic	200,000	1 x 3/8	1.03
	200,000	1/8 x 3/8	1.22
Al.	10×10^6	1/16 x 3/8	1.01

How the total stiffening effect of the type A-1, SR-4 gage is divided between the wire grid, which is the strain sensitive element, and the material which joins that grid to the strained surface was also investigated. The total cross sectional area of the six lengths of one mil wire in the grid is

$$A_w = \frac{(6)(\pi)(0.001)^2}{4} = 4.71 \times 10^{-6} \text{ in}^2.$$

If the modulus of elasticity of the grid wire (Advance) is taken as

$$E_w = 26 \times 10^6 \text{ psi}$$

then the force required to strain the grid is

$$\frac{F_w}{e} = A_w E_w = 122 \text{ lb/in/in},$$

in which the subscript $\underline{w}$ denotes the grid wire. Comparison of this value with the value of F/e for the type A-1 gage as a whole shows that the wire grid accounts for only

$$\frac{122}{1040} \text{ or } 12 \text{ percent}$$

of the total stiffening effect of the gage.

DESIGN OF A BONDED WIRE RESISTANCE GAGE FOR USE ON RUBBER AND RUBBER-LIKE MATERIALS

Though the wire grid in itself provides considerable reinforcing effect, the author believed that it would be worth while to investigate bonding agents having a modulus of elasticity in the range of rubber or rubber-like materials ($500 \leq E \leq 5000$); since by using such bonding agents the reinforcing effect of the gage could be greatly reduced. For this purpose a grid of one mil "Advance" wire was wound in the pattern sketched in figure 6, and this grid was bonded directly to the surface of an aluminum bar with Cat's Paw

Cement.* The grid was calibrated by loading the aluminum bar as a cantilever beam, measuring the deflection with a dial indicator, and calculating the strain at the gage point. The gage factor (G.F.) was found to be 1.04. The gage factor for a grid of Advance wire wound in this pattern on aluminum can be computed using the method given by Meier.[1] The value thus obtained is

$$G.F._{\text{Computed}} = 2.06.$$

The difference between the observed value and this theoretical value is apparently due to shearing strain in the bond between the strained surface and the strain sensitive wire gird (see figure 7). This shearing strain in the bond will tend to reduce the reinforcing effect of the gage, but at the same time gage sensitivity will be reduced. This decrease in gage sensitivity due to shearing strain in the bond appears to be the major difficulty in reducing the over-all stiffness of a bonded wire resistance gage by using a bonding agent which has a low modulus of elasticity. Additional experimental work has shown that, as the thickness of the "low modulus" bond is increased, the amount of strain reduction is increased; that is, the gage factor is decreased. This suggests the use of thin sheets of rubber between SR-4 gages and a strained surface to make a gage capable of measuring very large strain.

The conclusion that a wire grid is not the ideal resistive type transducer for use on rubber or rubber-like materials seems inescapable. However, the author has had some success in measuring transient, longitudinal strain pulses in rubber rods using a wire grid of the form shown in figure 6 bonded directly to the rubber rod with a rubber cement. Because the thickness of the bond is critical, these gages must be calibrated after mounting on the rubber rod; and this is a serious shortcoming.

SUMMARY AND CONCLUSION

Although bonded resistance type strain gages are well developed, their use for measuring strains in materials having low moduli of elasticity introduces some new problems. The conventional bonded wire resistance gage reinforces the surface of the test specimen at the point where the gage is attached so that the strain is reduced and the strain pattern is altered. This reinforcing effect is primarily due to the relatively high modulus of elasticity of the cement used to bond the wire grid to the strained surface. A strain sensitive wire grid can be bonded to the test specimen with a cement having a low value of modulus, and the reinforcing effect will be greatly reduced; however the gage sensitivity is likewise reduced. As long as a wire grid is used, some reinforcing effect is present, and this may be important.

For these reasons the use of some other strain-sensitive material seems desirable. In spite of the difficulties encountered in the past in attempts to use so-called conductive rubber as a strain transducer, its re-investigation now seems worth while, since it has high sensitivity and a low reinforcing effect.

REFERENCES

1. Meier, J.H. Strain rosettes, in Handbook of Experimental Stress Analysis, ed. M. Hetenyi, p. 407-9, John Wiley and Sons, Inc. New York.

*Cat's Paw Cement is a rubber cement available at almost any shoe repair shop.

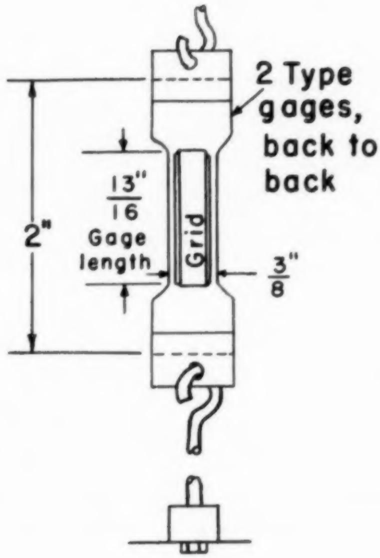

Figure 1. Cross Section of Flat Grid Type, Paper Base, SR-4 Gage.

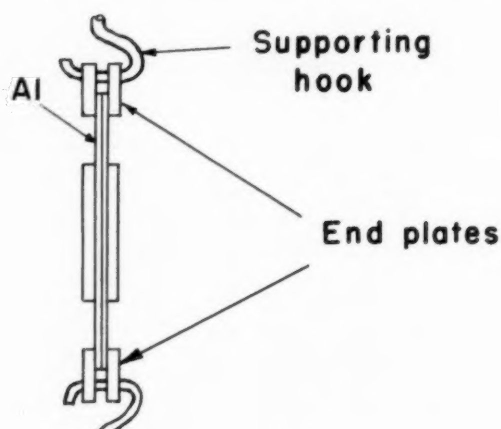

Figure 2. Two Type A-1, SR-4 Gages for Testing in Tension.

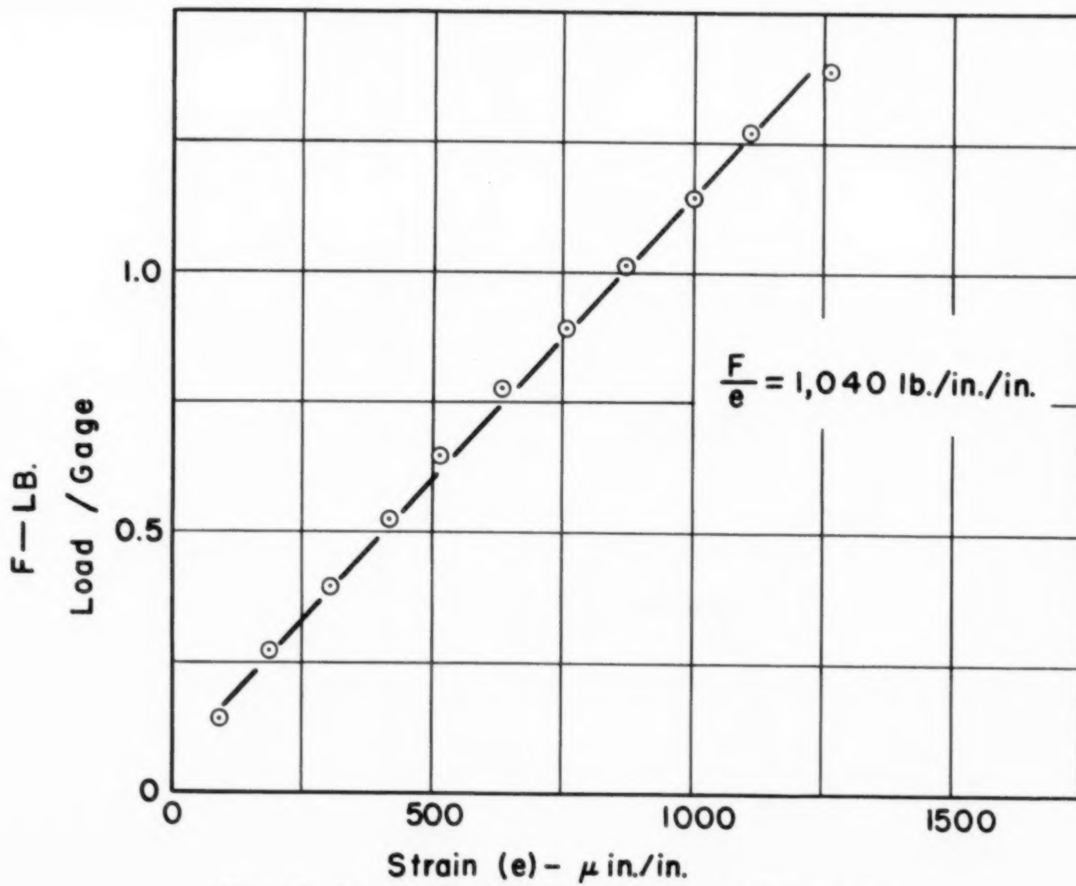

Figure 3. Load vs. Strain; Type A-1, SR-4 Gage in Tension.

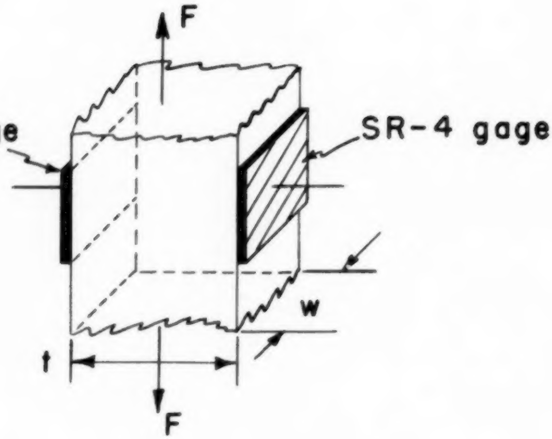

Figure 4. Two SR-4 Gages Reinforcing a Tensile Test Specimen.

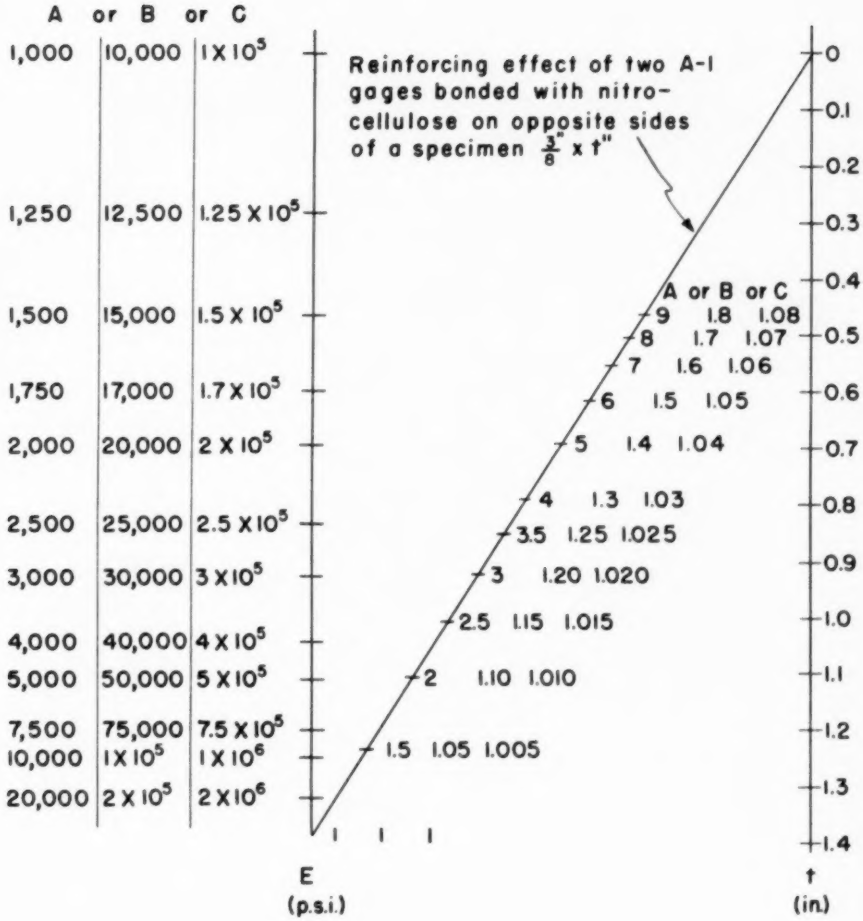

Figure 5. Nomograph for the Solution of Equation (4).

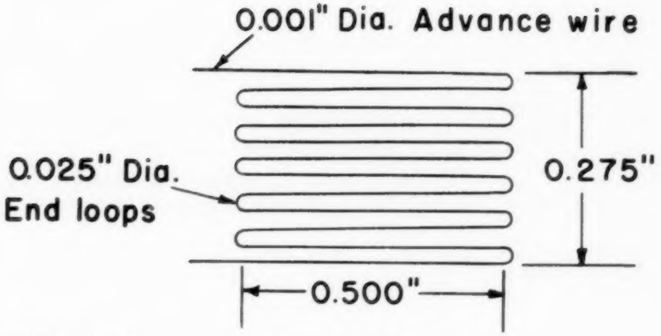

Figure 6. Wire Grid Pattern.

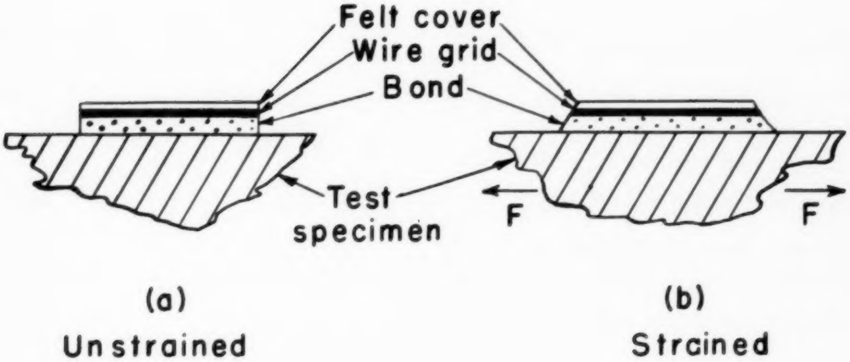

Figure 7.

Longitudinal Cross Section through a Bonded Wire Strain Gage, Showing Effect of Shear Strain in the Bond.